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# THE FLOW OF GAS IN A CHANNEL WITH A FLEXIBLE SKIRT

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The flow of perfect incompressible fluid in a channel bounded at its outlet by a flexible skirt in the form of an inflatable bag is considered. The bag material is assumed absolutely flexible, nonexpandable and weightless; pressure inside the bag is taken to be equal to that in the channel. The assumption that the flow in the proximity of the outlet minimum cross section is one-dimensional makes it possible to eliminate pressure when simultaneously solving the equations of fluid motion and of bag equilibrium. The derived differential equation for bag ordinates is solved numerically; for mildly sloping bags a solution of closed form is obtained with the use of elliptic integrals. The applicability range of the mildly sloping bag is determined by comparing results of numerical and approximate calculations. The derived pattern of pressure distribution along the bag wall is in agreement with experimental data [1].

Flexible elastic skirts are widely used in modern transport equipment supported on air cushions. Such skirts fixed at the cushion chamber periphery make it possible to contain a considerable volume of air in the chamber and to overcome obstacles in the path of the equipment. Elastic inflatable bags fixed to the bottom of the equipment along its periphery and connected by a passage to the air in the cushion chamber belong to such structures(*).

The aim of the present investigation is the determination of the outlet cross section, of the bag geometry, and of pressure distribution along its contour in terms of flow parameters.

Shape of the bag under working conditions is usually calculated on assumptions as follows [2]. Material of the bag is absolutely flexible but nonexpandable; the skirt is weightless; pressure inside the bag is constant and equal to the pressure in the cushion chamber when the two are connected by an air duct. The plane flow of perfect incompressible fluid is considered, i.e. the skirt is cylindrical with its axis normal to the direction of flow at infinity. These assumptions make it possible to reduce the problem to the investigation of the equilibrium of a flexible nonexpandable bag in a stream of gas

$$p_1 - p(x) = T_0 K \tag{1}$$

where $K$ is the bag contour curvature, $T_0$ is the tension in bag walls, $p_1$ the pressure in the latter, and $p(x)$ is the pressure distribution in the outlet gap in the longitudinal direction.

The described flow is schematically represented in Fig. 1. The bag is attached to

*) USA patent № 3291237, class 180-7, Dec. 13, 1966.

the equipment body at point $A$. Since the outlet gap is small and the bag is connected to the cushion chamber by an air duct, pressure in the bag $p_1$ and in the air chamber $p_0$ are balanced along the fairly large part $AC$ of the bag wall which is a straight line inclined to the horizon at angle $\nu$. At the end point B of the bag curved section, whose shape is determined below, flow separation takes place. Point $B$ is assumed sufficiently distant from point $C$ as compared to the height of the outlet gap, so as to make possible the assumption of one-dimensional gas flow in every channel cross section between the solid surface and curve $CB$.

To the right of the separation point $B$ the pressure drop $\Delta p = p_1 - p_a$, where $p_a$ is the atmospheric pressure, and as follows from Eq. (1.1), the part of the bag to the right of point $B$ is of circular shape. We call that part of the bag passive, as opposed to the active part $ACB$ in contact with the flow.

Tension of the bag passive part is determined by its radius—the inverse of curvature—by formula

$$T_0 = R\Delta p \tag{2}$$

Denoting the velocity, pressure and the height of the gap by $v(x)$, $p(x)$, $h(x)$, respectively, at any section of the channel between $CC_1$ and $BB_1$, from the Bernoulli equation and the condition of continuity

$$\frac{v_0^2}{2} + \frac{p_0}{\rho} = \frac{v^2}{2} + \frac{p(x)}{\rho}, \quad Q = v_0 H_0 = vh(x)$$

we obtain the equation

$$p_1 - p(x) = -\beta\left[\frac{1}{H_0^2} - \frac{1}{h^2(x)}\right] \quad \left(\beta = \frac{1}{2}\rho, \quad Q^2 = \text{const}\right) \tag{3}$$

We introduce the system of coordinates shown in Fig. 1, and using the expression for curvature in terms of $h$ and $x$, from Eqs. (1) and (3) we obtain

$$T_0 \frac{d^2h}{dx^2}\left[1 + \left(\frac{dh}{dx}\right)^2\right]^{-3/2} = -\beta\left[\frac{1}{H_0^2} - \frac{1}{h^2(x)}\right] \tag{4}$$

Multiplying both sides of Eq. (4) by $dh/dx$ and integrating, we obtain

$$-T_0\left[1 + \left(\frac{dh}{dx}\right)^2\right]^{-1/2} = -\beta\left[\frac{h(x)}{H_0^2} + \frac{1}{h(x)}\right] + C \tag{5}$$

where the constant $C$ is determined by the boundary condition

$$x = 0, \quad h = h_0, \quad dh/dx = -\operatorname{tg}\gamma \tag{6}$$

From (5) and (6) we have the equation for numerical solution

$$\frac{dh}{dx} = \pm\left\{\left[-\beta\left(\frac{h(x)}{H_0^2} + \frac{1}{h(x)}\right) - \frac{T_0}{\sqrt{1+\operatorname{tg}^2\gamma}} + \beta\left(\frac{h_0}{H_0^2} + \frac{1}{h_0}\right)\right]^{-2} T_0^2 - 1\right\}^{1/2} \tag{7}$$

The minus sign at the radical relates to section $CO$ where the angle of the tangent to the contour is negative and the plus sign is taken for section $OB$.

We solve Eq. (7) by Runge-Kutta method with the boundary condition $h = h_0$ at $x = 0$. The plus sign at the radical in (7) is maintained until the radicand remains positive, after which it is substituted by the plus sign. This means that the shape of the bag is first determined along section $CO$ up to point $O$ where the gap has its minimum, i.e. $dh / dx = 0$, and then along section $OB$ where $dh / dx > 0$.

Simultaneously with the solution of Eq. (7) we determine at each point with coordinates $x$, $h(x)$ with the use of Eq. (3) the pressure difference

$$p - p_a = \Delta p + \frac{\beta}{H_0^2}\left[1 - \frac{H_0^2}{h^2(x)}\right] \quad (\Delta p = p_1 - p_a) \tag{8}$$

We solve Eq. (7) until the pressure drop $p(x) - p_a$ which became negative in the vicinity of point $O$ of minimum gap, becomes zero at point $B$ where flow separation takes place. The tension $T_0$ in Eq. (7) is determined by formula (2).

Formulas (2), (7), and (8) show that the solution of Eq. (7) depends on a considerable number of input quantities: the flow rate $Q$, pressure drop $\Delta p$ and parameters which define the shape of the bag: the angle $\gamma$ of its inclination at point $C$, the distance $H_0$ between the equipment body and the horizontal surface, height $h_0$ of the point at which the bag becomes curvilinear, and of radius $R$ of curvature of the bag passive part. All parameters were chosen so that pressure $p(x)$ would not become negative along the curved section $CB$.

The calculation results are shown in Figs. 2—4. The shape of the bag shown in Fig. 2 was determined for the following parameters: $\Delta p = 60\,\mathrm{kg/m^2}$, $Q = 0.4\,\mathrm{m^3/sec}$, $R = 21\,\mathrm{cm}$, $H_0 = 30\,\mathrm{cm}$, $h = 5\,\mathrm{cm}$, $\gamma = -20°$, and the minimum gap $h_{min} = 1.08\,\mathrm{cm}$. Coordinates $x$ and $y$ in Fig. 2 are given in cm. The variation of pressure $p(x) - p_a$ along the bag is shown in Fig. 3 in $\mathrm{kg/m^2}$, where curve *1* relates to the same parameters as Fig. 2, and curve *2* corresponds to the following parameters: $\Delta p = 100\,\mathrm{kg/m^2}$, $Q = 0.7\,\mathrm{m^3/sec}$, $R = 28\,\mathrm{cm}$, $H_0 = 50\,\mathrm{cm}$, $h = 10\,\mathrm{cm}$, $\gamma = -20°$, and the minimum gap $h_{min} = 1.53\,\mathrm{cm}$.

It is seen that in the vicinity of point $O$, where the gap has its minimum an additional rarefaction is present in the flow. Along a small section pressure $p(x)$ falls below atmospheric, then increases, and at the separation point becomes equal to atmospheric. These results are in agreement with those obtained experimentally in [1].

The solution of problem (4), (6) can be obtained for small angles $\gamma$ in the form of elliptic integrals. By expressing the second factor in Eq. (5) in the approximate form $1 - {}^1/_2\,(dh/dx)^2$ and taking into account boundary condition (6), instead of (7) we obtain the following equation:

$$\frac{dh}{dx} = -\frac{1}{H_0}\sqrt{\frac{2\beta}{T_0}}\left[-h(x) - \frac{H_0^2}{h(x)} + F\right]^{1/2} \tag{9}$$

$$F = \frac{H_0^2 T_0}{2\beta}\,\mathrm{tg}^2\,\gamma + h_0 + \frac{H_0^2}{h_0}$$

where the minus sign shows that it defines the shape of the flexible contour along section $CO$ (Fig. 1) up to point $O$, where the angle of the tangent to the contour increases from its initial value $-\gamma$ to zero.

Substituting $\tau = \sqrt{h}$ into (9) and integrating it with boundary conditions $h = h_0$ at $x = 0$, we obtain

$$x = H_0\sqrt{\frac{2T_0}{\beta}}\int_{\sqrt{h}}^{\sqrt{h_0}} \frac{\tau^2 d\tau}{\sqrt{(a^2 - \tau^2)(\tau^2 - c^2)}} \tag{10}$$

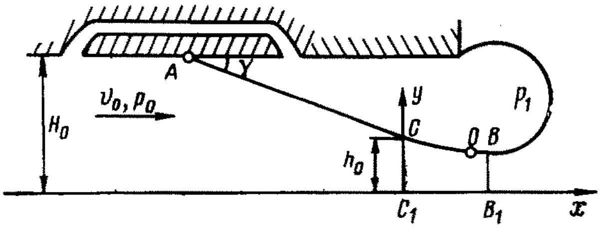


Fig. 1

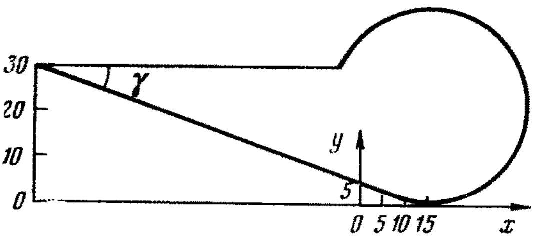


Fig. 2

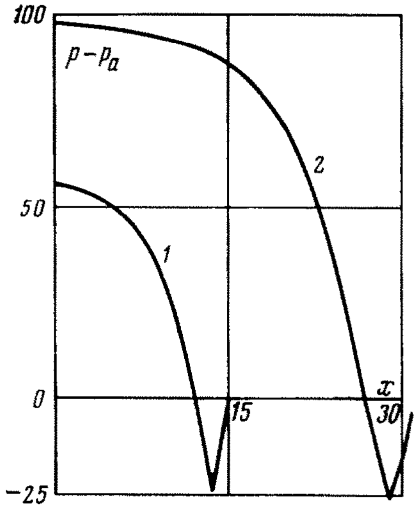


Fig. 3

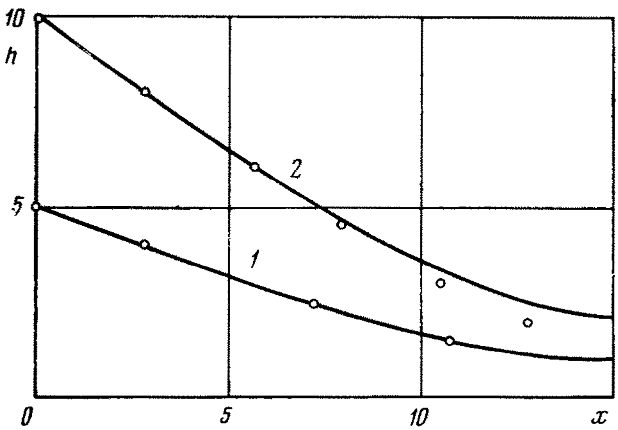


Fig. 4

$$a=\sqrt{\frac{F}{2}+\sqrt{\left(\frac{F}{2}\right)^2-H_0^2}}, \quad c=\sqrt{\frac{F}{2}-\sqrt{\left(\frac{F}{2}\right)^2-H_0^2}}$$

$$(0<c<a)$$

By substituting $x^2=(a^2-\tau^2)/(a^2-c^2)$ we can test the equality

$$\int_{\sqrt{h}}^{a}\frac{\tau^2 d\tau}{\sqrt{(a^2-\tau^2)(\tau^2-c^2)}}=aE(\lambda, q) \quad (0<c<\sqrt{h}<a)$$

$$\lambda=\lambda(h)=\arcsin\sqrt{\frac{a^2-h}{a^2-c^2}}, \quad q=\frac{\sqrt{a^2-c^2}}{a}$$

which is used for computing the integral in (10), and where $E(\lambda, q)$ is an elliptic integral of the second kind.

If the gap $h$ and its limit value $h_0$ satisfy the condition $0<c<\sqrt{h}<\sqrt{h_0}<a$, formula (10) can be written as

$$x=H_0 a\sqrt{\frac{2T_0}{\beta}}\,[E(\lambda(h), q)-E(\lambda(h_0), q)]$$

The shape of the bag in the vicinity of the minimum outlet gap is shown in Fig. 4 in the form of curves *1* and *2* determined by the numerical solution of Eq.(7). The small circles relate to the approximate solution obtained with elliptic integrals.

Curve *1* in Fig. 4 relates to the same parameters as in Fig. 2 ($\gamma=-20°$); curve *2* corresponds to the following values of parameters: $\Delta p=60\,\mathrm{kg/m^2}$, $Q=0.9\,\mathrm{m^3/sec}$, $R=15\,\mathrm{cm}$, $H_0=30\,\mathrm{cm}$, $h=10\,\mathrm{cm}$, $\gamma=-36°$, and the minimum gap $h_{min}=2.2\,\mathrm{cm}$. It will be seen that the numerical and the approximate solutions coincide for $\gamma=-20°$ and diverge for $\gamma=-36°$.

## REFERENCES

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